विबुधाभिवन्दितपदो जयति श्रीभास्कराचार्यः

International Conference on the 900th Birth Anniversary of Bhāskarācārya

19, 20, 21 September 2014

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I BHĀSKARĀCĀRYA'S LIFE AND TIMES

Learning and Patronage in 12th -13th Century A.D. Bhaskarācarya and the Śāndilya Family, A Case Study

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Bhāskarācārya, an eminent mathematician and astronomer- sometimes described as one of the most noted ten astronomers of the ancient world - contributed to Indian astronomy in a most prolific manner. Born in Śaka 1036, he composed a number of treatises, and left a lasting impression on posterity and legacy in the form of original contribution to the sciences of astronomy and mathematics. Within a couple of generations after him, the royal family at Pattanapura of the Yādava period (present Patne in the neighborhood of Chalisgaon in Jalgaon district) felt the need of establishing a school (*maṭha*) in his memory to study and teach the treatises written by him. His own son Lakṣmīdhara, who was a pundit in the court of the local Nikumbha rulers, was invited as the Chief Astronomer by Yādava Jaitrpāla. His gradson, Caṃgadeva, also was similarly honoured in A. D. 1207, by Yādava Siṃghana (II) who had established in an imperial position.

The story does not end here. Caṃgadeva's cousin, Anatadeva, grandson of Śripati who was a brother of the illustrious Bhāskarācārya, succeeded him in the imperial court of Siṃghana. Anantadeva's brother Maheśvara was the composer of the inscription at Bahal, not far away from Patne in the same taluk of Chalisgaon; this inscription recounts about the literary and scientific contributions of this lateral family. At Balasane, in the nearby district of Dhulia was a similar centre for learning, *rājamaṭha*, repaired by one Mahallūka Paṇḍita, who has been described as 'the very Sun to the discipline of Mathematics'.

There are certain questions that have been raised by the scrutiny of these inscriptions found in the Khandesh area:

i) at least three centres of learning , within a perimeter of 100 kilometers , that specialise in the study of mathematics and also astronomy , one of them especially to study the works of Bhāskarācārya ;

ii) a very complex system of providing support, in kind and in cash, was devised, wherein the Brahmins , association of traders, the municipal authorities, the local chieftain and the king/ sovereign came to provide funds;

iii) the period of 300 years (c. A. D. 915 to 1244) covered by the nine generations of the learned family of Bhāskarācārya , as the literary and inscriptional sources tell us, must have moved from Nausari (in modern Gujarat) to Ujjain/ Dhar (in modern Madhya Pradesh) to Bijjala-bida (Beed in present day Marathwada of erstwhile Nizam's

Dominions/ Maharashtra) and finally settled at Patne / Bahal, under the patronage of the Chālukyas and the Yādavas.

This illustrious family well versed in various branches of Vedic learning and traditional scholarship, producing gifted men of high literary achievements, and equally proficient in abstract sciences contributed profusely in different walks of life. If we just take into consideration the political career of Bhoja Paramāra the patron of Bhāskarabhaṭṭa, son of poet Trivikrama, the first known ancestor in the line, one is amazed to notice the political upheavals turmoil undergone by subjects and residents of Malwa, Bundelkhand, southern Gujarat and the Deccan (including the dominions of the Kākatīyas of Warangal, Chālukyas of Kalyāṇa, Hoyasalas of Dvārasamudra and the Yādavas of Devagiri). Generally it is said the great works of art and science are produced in times of peace and leisure; the lives of Bhāskarācārya and scions of his family defy this maxim. Similarly no one could have believed of the great political transformations and catastrophes that shook the cultural life within a century or so after the demise of the great astronomer-mathematician.

II BHASKARĀCĀRYA'S POETIC GENIUS

The poetical face of the mathematical and astronomical works of Bhāskarācārya

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In the twelfth century Sanskrit poetry has reached the zenith of its refinement. Bhāsakarācārya, born in 1114 A. D., has been a pandit of his times and has taken a place among the luminaries of this period of glory of Sanskrit humanities. We owe to a grandson of Bhāskara, Caṅgadeva, a detailed picture of the tradition of *pāṇḍitya* of his family in an incription reporting the foundation of a school for teaching the works of his grandfather. Bhāskara was an accomplished *paṇḍita* well-versed in *Veda*, two schools of *mīmāṃsā*, *sāṃkhya*, *vaiśeṣika*, *tantra*, *śilpa*, *chandas*, *kāvya* and the three branches of *jyotiṣa (saṃhitā, gaṇita, hora)*.

We can differentiate two styles in the four parts of the *Siddhāntaśiromaṇi*, *Līlāvatī*, *Bījagaṇita*, *Grahagaṇitādhyāya* and *Golādhyāya*. There is an appropriate style for the expression of the mathematical facts, which can be called *sūtra* style. There is another style for the examples, which we will call *udāharaṇa* style. The latter is characterised by its freedom from the conciseness of the *gaṇitaśāstra*, giving space for extra matter and wide scope to imagination. There, we find the famous poetical problems implicating themes of Sanskrit poetry, the bee on the lotus suggesting the *śṛṅgāra-rasa*, the feats of Arjuna bringing *vīra-rasa* etc.

The *sūtra* style aims at clarity. It obeys the śāstric habit of conciseness, without excess to preserve clarity. It is not devoid of literary qualities, musicality of well-masterd prosody and choice vocabulary, with a pleasant use of *bhūtasaṃkhya*-s and numerous alliterations. *Kāvyālaṃkāra*-s are frequently used. Qualities of *kāvya* are noticeable, such as *prasāda*.

We have also to examine differences of style and treatment of contents in the four parts of *Siddhāntaśiromaṇi*. The *Golādhyāya* appears more open to extraneous inspiration such as purāṇic conceptions, and accepts more metaphoric images, than other *adhyāya*-s. Moreover reflection on some mathematical propositions of Bhāskara leads to a question: is poetical inspiration at the origin of some scientific ideas? This can be considered in the case of the division by zero, in visions of the universe in the *Bhuvanakośa* of *Golādhyāya* etc. Noteworthy is the fact that Bhāskara has stated his feeling of wonder before phenomena of nature: *vicitrā bata vastuśaktayaḥ,* "wonderful indeed are the powers of things".

On these themes the proposed paper will give many examples and bring in comparison with formulations of other authors, in order to show clearly how Bhāskara is a literary exception in *jyotiṣa* literature.

Bhāskarācārya as a Poet

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Among the classical literature, Sanskrit contains largest number of compositions in poetical and prosaic form. Even the texts on philosophy, religion, science, architecture, mathematics, astronomy, astrology, medicine, fine arts like music and dance and so on were mostly written in poetry, so that it would be easier for men to memorize them, keep them intact and pass on the same to posterity. Also, the writers on these subjects found it easy to express their views freely through verses, at the same time exhibit their skill in language and literature. Thus we find thousands of texts written in poetic form on various subjects in Sanskrit.

The period before 11th century A.D. produced great poets like Kālidāsa, Daṇḍin, Bhāravi, Māgha and so on. During 9-11 Cent. A.D. even at the advent of foreign invasion, many poets kept the Indian culture intact through their creations. For example, Rājanaka Ratnākara, Kalhaṇa and Kṣemendara of Kashmir, Halāyudha, Hemacandra, Śrīpāla, Bilhaṇa (of Kashmir origin, but who adorned the court of King of Kalyān), and others dominated the country with their contribution in poetry.

Another giant belonging to the 12th Cent. A.D. is Bhāskara II from Mahāraṣṭra who inherited the poetic skill due to the influence of his predecessors in this field. His contribution in the field of Mathematics has been studied by modern Indian and foreign scholars in the field of mathematics. It would also be interesting to read his works from literary point of view.

Bhāskara, in his *Siddhānta Śiromaṇi* consisting of *Golādhyāya, Grahagaṇita, Līlāvatī* and *Bījagaṇitam* exhibits his deep knowledge in composing verses in many metres. His knowledge in *Chandas Śāstra*, *vyākaraṇa, alaṅkāra* while using both *śabda* and *arthālaṅkāras, subhāṣitas*, his insight into various schools of *darśanas*, his acquaintance with Itihāsas and Purāṇas, the references of many cities and countries which manifest his knowledge in Geography, good grip of *kośa* literature in Sanskrit as he uses many words to denote one object — all these show that Bhāskara II deserves to be appreciated as a good poet.

This paper presents a picture of Bhāskara II as an able *kavi* in the light of above facts.

Contribution of *Līlāvatī* to Prosody

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Līlāvatī, a treatise on Mathematics, is written by Bhāskara II who lived in 12th century AD. Besides explaining the details of mathematical concepts that was in existence up to that period, the text introduces some new mathematical concepts. This paper is an attempt to analyze the metres employed in ***Līlāvatī*** as well as the concepts of permutation and combinations introduced by the author.

In the last line of the benedictory verse, the author reveals that ***Līlāvatī*** is a composition made concise and simple with charming words. In this verse the author employed the 19-syllabled *śārdūlavikrīdita*. But the benedictory verse of the first sub-chapter is in *Anuşţubh* metre. For the *Karaṇasūtra*-s, the author has employed various metres like *Anuşţubh, Indravajrā, Upendravajrā* and *Upajāti*. The verses cited as examples reveal the poetic skill of the author. The metre *Sragdharā* which is included in *Prakṛti* group and which is composed of seven trisyllabic *gaṇa*-s is profusely used by the author. Besides these, the author employs a lot of other metres like *Drutavilambita, śikhariṇī, śārdūlavikrīdita, Vasantatilaka, Mālinī* etc. to make the verses melodious to hear. These rhythmic compositions make the learners more interested in further studies.

In this text, mathematical concepts related to metres also have been discussed. In prosody, permutation is a mathematical calculation that gives the possible number of metres in a given *Chandas*. In the 10th sub-chapter entitled *Chandaścityādi,* the author has introduced mathematical formula for calculating the permutation of every metre. In the *Karaṇasūtra* 57, the author says that the number of syllables in every metre is named as '*Gaccha*' (*pādākṣaramita gaccha*). 8 is the *Gaccha* in the eight syllabled *Anuṣṭubh*. The number of permutations in this metre is 2^8. Here '2' denotes the basic constituence i.e. *Guru* and *Laghu* and the 8 denotes the number of syllables or *Gaccha*. If the *Gaccha* of a *sama* metre is 'n' the the number of permutation is 2^n.

For example - In an 8 syllabled *Anuşţubh* the permutation is 2^8 i.e., 256. In an 8 syllabled *Ardhasama* metre, the permutation is $2^{2n} - 2^n$, i.e., $2^{16} - 2^8$ (= 65280). The permutation of *Vi1ama* metre $2^{4n} - 2^{2n}$, i.e., $2^{32} - 2^{16}$ (=4294901760).

In the *Karaṇasūtra* 59, 60 and 61 are related to 'combination' the author narrates how to find out the number of metres on the basis of the number of *Guru*-s from permutation.

The present paper proposes to discuss the principles of permutations and combinations employed in this section of ***Līlāvatī***, in particular.

लीलावती में काव्यसौंदर्य

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भारतीय संस्कृति का मूलाधार वेद हैं। भारतीय विद्याएँ वेदों से ही प्रकट हुई हैं। वेदों के छः अंग कहे गए हैं- (1)शिक्षा(2)कल्प(3)व्याकरण(4)निरुक्त(5)छन्द और(6)ज्योतिष। इन्हें षड् वेदांगों की संज्ञा दी गई है। अनुष्ठानों के उचित काल-निर्णय के लिए ज्योतिष का उपयोग मान्य है।महर्षि पाणिनि ने ज्योतिष को वेद पुरुष का नेत्र कहा है-`ज्योतिषामयनं चक्षुः'। त्रिस्कंध ज्योतिष शास्त्र में सिद्धान्त के अपर पर्याय के रूप में गणित शास्त्र का व्याख्यान किया गया है। `लीलावती' ग्रंथ आचार्य भास्कर द्वारा ग्रथित गणित मणिमाला की एक मणि है। भास्कराचार्य ने इस लघु ग्रंथ में गहन गणित शास्त्र को अत्यंत सरस ढंग से प्रस्तुत कर गागर में सागर की उक्ति को प्रत्यक्षतः चरितार्थ किया है। इकाई आदि अंकों के परिचय से आरम्भ कर अंकपाश तक की गणित में प्रायः सभी प्रमुख एवं व्यावहारिक विषयों का सफलतापूर्वक समावेश किया गया है। जहाँ एक तरफ इतने गूढ़ प्रश्नों का विवेचन है वहीं दूसरी तरफ भास्कर की ललित पदावली स्वर्ण में सुगन्ध का कार्य करती है। लीलावती की सरस छन्दोमयी भाषा पाठकों को गणित की तरफ अनायास ही आकृष्ट करती है। गणित के कुछ उदाहरणों में भास्कर के सरस कवि हृदय का स्पन्दन स्पष्ट रूप से लक्षित होता है। श्रंगार रस से ओत-प्रोत गणित के उदाहरण किसी को भी गणित की ओर बलात आकृष्ट करने में समर्थ हैं। प्रस्तुत शोध-पत्र में भास्कर विरचित काव्य लीलावती में वर्णित छन्द-सौन्दर्य,अलङ्कार-विधान,रस-परिपाक,प्रकृति-चित्रण,पर्यायवाची शब्दों का गुम्फन,लोक-शिक्षा,नीति-शिक्षा आदि के आधार पर काव्यगत विशेषताओं का वर्णन किया गया है तथा यह सिद्ध करने का प्रयास किया गया है कि कैसे गणित जैसे गम्भीर विषय का अध्ययन भी सरस काव्यमयी भाषा एवं श्रंगारिक भाव के कारण पाठक के मन को बोझिल नहीं होने देता।

III. THE LĪLĀVATĪ

The Līlā of the Līlāvatī
A Beautiful Blend of Arithmetic, Geometry and Poetry

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It is well known that the *Līlāvatī* of Bhāskarācārya has been the most preferred text to introduce mathematics in India for more than seven centuries since its composition in the twelfth century. Though this text has been more or less completely ignored in the modern system of education, it is still employed in the traditional schools and colleges (*pāṭhaśālās*) to teach the basics of arithmetic and geometry.

Anyone who reads the *Līlāvatī* wouldn't take time to get convinced as to why it has been the most favoured choice to introduce mathematics. Starting from the basics, namely the units of measurement, the text~~s~~ covers a wide range of topics that includes discussion on fundamental arithmetical operations, application of the rule of three, operations with series, principles of geometry, finding areas of planar figures, solving first order indeterminate equations, and so on. The most striking feature of the *Līlāvatī* lies in interspersing the *sūtras* (verses that present a formula or enunciate a mathematical principle) with several appealing examples drawn from day to day life. The wonderful choice of metres --- *anuṣṭubh, indravajrā, āryā, vasantatilakā, śārdūlavikrīḍita*, etc. --- made by Bhāskara to aptly depict the theme of the problem makes the reading all the more interesting and quite enjoyable. No wonder Śaṅkara Vāriyar in his commentary *Kriyākramakarī* extolled the text *Līlāvatī* as *vṛttiratnam* (a gloss par excellence).

During the lecture we would attempt to highlight not only the mathematics in the *Līlāvatī,* but also the felicity and grace with which it has been rendered in the form of beautiful poetry by Bhāskara by blending it with a variety of metres, the teasing double meanings, and so on. In short we will try to bring out the *līlā* in the *Līlāvatī*.

A Comparative Study of Bhāskarācārya's Līlāvati and Mahāvīrācārya's Gaṇitasārasaṁgraha

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Bhāskarācārya or Bhāskara II the great poet and mathematician was born in 1114 A.D. He belonged to Śāṇḍilya lineage. His master was his own father Maheśvara who was a great astrologer. At the age of 36, Bhāskarācārya wrote Siddhāṁtaśiromaṇi which consists of four parts one of which is Līlāvati. It mainly deals with Arithmetic but also contains Geometry, Trigonometry and Algebra. Līlāvati provides prerequistes for the study of planetary motions and Astronomy. It also contains solutions of Diophantine Equations.

Mahāvīrācārya was a famous mathematician from the Jaina School who is counted among the well-known scholars Āryabhaṭa, Varāhamihira and Brahmagupta. Not much is known about his personal life. According to the literature available he hailed from Karnataka and enjoyed the patronage of the Rāshtrakūṭa king Amoghavarsha Nṛpatunga who ruled Mānyakheṭa (now Maḷakheḍa, Gulbarga District, Karnataka State).

Mahāvīrācārya was the author of the Sanskrit work Gaṇitasārasaṁgraha (= GSS). GSS is a collection of mathematics of his time. GSS was used as a text for several centuries in Southern-India to teach mathematics for children. Thus GSS became very famous as well as popular.

The object of this paper is to make a comparative study of Līlāvati and Gaṇitasārasaṁgraha.

Gaṇeśa Daivajña's Upapattis on Līlāvatī

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Gaṇeśa Daivajña of 16th century AD is a prolific writer with mathematical, astrological, astronomical, metrical and Dharma-śāstra works to his credit, both original texts and commentaries. Of his commentaries, the *Buddhivilāsinī* on Bhāskara's *Līlāvatī* (*L*) is very much appreciated for the *upapattis* it provides for most of the rules and solutions of *Līlāvatī.* Gaṇeśa's *upapattis* are in the form of logical explanation, algebraic application, demonstration or activity and geometrical proof.

Līlāvatī gives a rule (*L.* 45) for operations related to zero. Gaṇeśa gives logical explanation for the rule, 'product of a number and zero is zero'.

A rule (*L.* 60) is given for solving a *vargakarma* problem {To find the two quantities x and y such that ($x^2 + y^2 – 1$) and ($x^2 – y^2 – 1$) yield square roots}. Bhāskara II gives three solutions for this problem and Gaṇeśa provides algebraic proof for the solutions.

Līlāvatī gives the rule (*L.* 201) : 'In a circle, a quarter of the diameter multiplied by its circumference is its area. Four times this area is the surface area of a sphere. The product of the surface area and diameter divided by six is the volume of the sphere'. Gaṇeśa explains the rule and gives interesting demonstrations in the case of the area of a circle and surface area of a sphere and an *upapatti* for the volume of the sphere.

The paper aims at explaining the *upapattis* mentioned above, a sample each of logical, algebraic and geometric proofs, as provided by Gaṇeśa Daivajña.

Parameswara's Unpublished Commentary on Lilavati
Perceptions and Problems

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Bhaskaracarya's *Lilavati* (1150CE) is the most celebrated work of Indian Mathematics and is still used as a textbook in Sanskrit institutions in India. The text, in fact, is one of the major portions of a greater work called *Siddahantashiromani*. In 266 verses, it deals with arithmetics, elementary algebra, mensuration and geometry. In the abundance of commentaries, it ranks with *Bhagavadgita* and *Kavyaprakasha* (which is a Sanskrit poetics work composed by a Kashmir scholar Mammata Bhatta on 11th century).

Among the commentaries, Parameswara's *Vivarana* (about 1430), Ganeshadaivajna's *Buddhivilasini* (1545), Suryadasa's *Ganitamrta* (c.1538), the gloss of Ranganatha on *Vasana* (c.17th cent.- *Vasana* is the demonstratory annotation of *Siddhantasiromani* composed by Bhaskaracharya himself) etc. are notable. According to R. C. Gupta, a well-known historian of Indian Mathematics, the best traditional commentary is the *Kriyakramakari* (C. 1534) of Shankara Variyar and Mahishamangalam Narayana (who compiled this after the demise of Shankara). It is worthy to note that even R. C. Gupta may not have considered the commentary of Parameswara while commenting such a statement; as this commentary was hidden in the form of unpublished palm leaf manuscripts. The present writer has unearthed three manuscripts of Parameswara's commentary - all written in legible Malayalam script.

In the present paper, for the first time in the history of Mathematics, an attempt is made to discuss the variant readings of Parameswara's commentary. It is beyond doubt that this attempt will be of much importance and an indispensable tool for an adequate understanding of the text *Lilavati*.

Līlāvatī and Kerala School of Mathematics

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Most of the historians thought that advancements in Indian mathematics came to an end by 12th century with Bhāskarācārya (b. 1114 A.D). Till recently historians were of the opinion that after 12th century no original contributions were made by Indians, rather they were only chewing the cud with the available works especially that of Āryabhaṭa and Bhāskara. Only recently when the details of works of Mādhava, Parameśvara, Dāmodara, Nīlakaṇṭha, Jyeṣṭhadeva, Acyuta piṣāraṭi etc. came to light, the above assumption was proved baseless. Now the contributions made by South Indians of medieval period (i.e., from 14th century to 17th century) are seriously studied and are being acknowledged globally. These contributions mainly came from the Kerala based astronomer-mathematicians who had a long traditions and continuous lineage (*guru-śiṣya paramparā*) from Vararuci (3rd century) to Sankaravarman(19th cent.). This tradition is now known as Kerala School of Mathematics and it is now widely accepted that this school surpassed its western counter parts at least by 200 years in discovering Infinite Series Expansions, Mathematical Analysis, Fundamentals of Calculus etc.

When we critically scrutinize the works of Kerala School, it could be seen that two ācārya-s, whom they refer to abundantly with high esteem, are Āryabhaṭa and Bhāskarācārya. Also texts ***Āryabhaṭīyam*** and ***Līlāvatī*** were also used as two fundamental text books to start learning the subject and further used as a standard reference. The most significant feature of Kerala School was that, all of the Astronomer-Mathematicians of Kerala School, critically analysed the ancient texts. They elaborated, corrected and supplemented to ideas in ancient texts after analysing the rationale behind it. A typical example of such a correction proposed for a verse in ***Līlāvatī*** as stated in ***Yuktibhāṣā*** (c 1530) is discussed in the proposed paper.

While discussing areas of quadrilaterals and triangles, Bhāskarācārya states in the ***Līlāvatī*** :

sarvadoryutidaḷam catusstitam bāhubhirvirahitam ca tadvadhāt/

mūlam asphuṭaphalam caturbuje spaṣṭamevam uditam tribāhuke//

The correction proposed in ***Yuktibhāṣā,*** for the second hemistich, is

mūlamatra niyataśrutau phalam tyarabāhujamapi spuṭam bhavet/

Malayalam Commentaries on *Līlāvatī*
A Survey of Manuscripts in Kerala

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The classical texts on Mathematics like ***Līlāvatī*** (1150 CE) assume relevance today only when we unravel the methodology and approach of the brain behind. This fact is strikingly so in the Indian context since the mathematical tradition of ancient and medieval India does not reveal the rationales and methodologies behind any of the findings.

It is well known that in the post-***Bhāskarācārya*** era of Indian Mathematics, Kerala was a centre of continuous and serious mathematical activity. This school of Mathematics, which, scholars have started naming as the **Kerala School of Mathematics**, produced a profusion of commentarial literature on Indian Mathematics. Prof. K. V. Sarma, who has unearthed and edited many of the valuable works produced by this School, has rightly pointed out that the most striking common feature of these works is the elaborate exposition of the rationales of many findings enunciated in the Classics of Indian Mathematics. The ***Kriyākramakarī*** commentary on ***Līlāvatī*** is a typical example for this. Hence commentaries on ***Līlāvatī*** produced by this school carry much significance.

Prof. K. V. Sarma has recorded that there are six commentaries on ***Līlāvatī*** written in Malayalam language (this is apart from the five commentaries in Sanskrit). Till date none of these has been printed. In his significant work ***A History of Kerala School of Hindu Astronomy***, Prof. Sarma has furnished the details of 16 manuscripts of Malayalam commentaries on ***Līlāvatī*** (most members of this list belong to the Oriental Research Institute and Manuscripts Library, Trivandrum). The present investigators have made a thorough survey of the available manuscripts of the commentaries, written in Malayalam language, on ***Līlāvatī*** in the three well known Public repositories of manuscripts in Kerala viz.

1) Oriental Research Institute and Manuscripts Library, Trivandrum,
2) Sree Ramavarma Government Sanskrit College Grantha Library, Trippunithura; and
3) Thunchan Manuscripts Repository, University of Calicut.

This paper will reveal the significant findings of the survey. The scope and feasibility of editing the manuscripts will also be discussed. The extent and style of the works will also be subjected to discussion.

Mensuration of Quadrilaterals in Lilavati

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Mensuration with quadrilaterals had received attention in the Siddhanta tradition at least since Brahmagupta. However in Bhaskaracarya's Lilavati we come across some distinctively new features. This talk will be an attempt to put the development historical perspective.

Aṅkapāśa in the Līlāvatī

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At the end of the *Līlāvatī*, Bhāskara added a new subject called *aṅkapāśa*. Here he gave four rules with examples and auto-commentary: (1) permutations of distinct digits such as 2, 1. This rule can be applied to different objects more than 9; (2) permutations of non-distinct digits such as 2, 2, 1, 1; (3) permutations of distinct digits such as digits from 1 to 9 in 6 places; (4) permutations of digits that add up to a specified sum in specified number of places such that five digits are placed in 5 places and the sum of the digits is 13. These rules are different from the rule for the number of combinations without repetition given in mixture (*miśra*) procedure.

About 200 years later Nārāyaṇa Paṇḍita provided many rules and examples in a chapter called *aṅkapāśa* in his *Gaṇitakaumudī*. Nārāyaṇa employed the terms used in the *pratyayas* of the *Chandaḥsūtra*, namely, *saṃkhyā* (number of variations), *prastāra, naṣṭa* and *uddiṣṭa*. Of these terms, Bhāskara used *saṃkhyā*, but he meant by it a number produced by a concatenation of digits like *saṃkhyāvibheda* (variation of numbers). He calculated *saṃkhyaikya* (sum of numbers) in the first two cases. At the end of the fourth rule Bhāskara said that only the compendium was told for fear of prolixity (*saṃkṣiptam uktaṃ pṛthutābhayena*). I wish to discuss what he meant by this statement. Moreover, I propose to investigate Bhāskara's sources for the *aṅkapāśa* and what later mathematicians learnt from Bhāskara.

Use of *Bhūta-Saṅkhyās* (Object Numerals) in *Līlāvatī* of Bhāskarācārya

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Bhūta-Saṅkhya system is a method of expressing numerals with specific words in Sanskrit. In this system words having symbolic meaning are used to connote numerical values. It was developed in India in the early centuries of the Christian era. The first nine numbers were denoted by words *eka, dvi, tri, catur, pañca, ṣaṭ, sapta, aṣṭa* and *nava* since *vedic* period. Several powers of ten were also mentioned by words. Zero was included later as part of the numeral system. But there was difficulty in denoting very big numbers in astronomical and mathematical texts, as they were composed in verse form. Hence this method of object or word numerals became popular among Indian astronomers and mathematicians for metrical suitability. The system has also been used in inscriptions and manuscripts for inscribing dates as well as in metrics. Familiar concepts from all branches of knowledge have been used for selecting the number-names so the system served the purpose of bringing together mathematics and culture too.

Various innovative methods of using *Bhūta-Saṅkhyās* are found in mathematical texts. Single words indicating a digit were commonly used but sometimes a single word denoted a two-digit number. Appropriate words were placed one after the other with place value to form long numbers and read from right to left as per the principle *aṅkānāṃ vāmato gatiḥ*. Sometimes a compound of object numerals and usual numerals was conveniently used. Sometimes multiple worded number-chronograms have been employed too.

Bhāskarācārya was a skilled writer and one of the striking features of *Līlāvatī* is the choice of precise and specific words. On this background this paper investigates how Bhāskarācārya has made use of *Bhūta-Saṅkhyās* in *Līlāvatī*.

The Legend of Līlāvatī

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Unlike the writers of other branches of Sanskrit learning about whom we know nothing but their names, authors of works on *Jyotiḥśāstra,* from Āryabhaṭa onwards, generally mention the years of their birth or epochs that are closer to their own times. In his *Siddhāntaśiromaṇi,* Bhāskarācārya mentions the year of his birth, the time of the completion of this work, the name of his father and his own place of residence. More important, he is the only astronomer for the propagation whose works a *maṭha* was

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established. Bhāskara's grandson Caṃgadeva, who set up the *maṭha,* recorded this fact in an inscription of 1207, where he also gives a detailed genealogy of his family.

Thus Bhāskara is not a mythical figure like Kālidāsa, but a historical personage whose date, provenance and authorship are firmly established. Even so, legends grew up about him that are perpetuated like the legends of Archimedes' bathtub or Isaac Newton's apple. The most persistent legend about Bhāskara is that he composed the *Līlāvatī* to console his widowed/unmarried daughter Līlāvatī.

In this paper we examine the veracity of this legend on the basis of internal evidence from Bhāskara's own works and external evidence from the commentaries on his works and contemporary social mores. We shall also discuss how the legend originated in the preface of the Persian translation of the *Līlāvatī* by Abū al-Fayḍ Fayḍī and how it was propagated in the West by Charles Hutton's *Tracts on Mathematical and Philosophical Subjects.*

IV THE BĪJAGAṆITA

Bījagaṇita of Bhāskara

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Mathematics, as everyone is aware, has a long tradition in ancient and medieval India. Starting from the Śulba Sūtras, Āryabhaṭa, Brahmagupta, Mahāvīra, Śrīpati, Śrīdhara and Padmanābha have contributed directly and indirectly to the growth of Algebra. Of these Bhāskaracārya of 12^{th} Century stands out for having treated Algebra as a separate genre of mathematics.

With the advent of Bhāskara, Indian mathematics reached the pinnacle of its achievement. This was the discovery of the method of solving the (erroneously called) Pell's equation $Nx^2 \pm 1 = y^2$, where N is a non-square positive integer and x and y are also required to be integers. This was known as the *cakravāla* method or cyclic method. To use Brahmagupta's method for the equation $Nx^2 \pm k = y^2$, there had to be an initial auxiliary equation which Brahmagupta could find only by trial and error method. Bhāskara, following his predecessors, achieved remarkable success when he evolved a simple and elegant method which helped to derive an auxiliary equation having the required interpolators k as $\pm 1, \pm 2, \pm 4$, simultaneously with two integral roots from another auxiliary equation empirically formed with any simple value of the interpolator, positive or negative. This method was called *cakravāla*, because "it proceeds as in a circle, the same set of operations being applied again and again in a continuous method".

In Bhāskara's *Bījagaṇita*, the initial chapters deal with *dhanarṇaṣaḍvidha* (law of signs), *Khaṣaḍvidha* (laws of zero and infinity), *avyaktaṣaḍvidha* (operations of unknowns), *karaṇī* (surds), *kuṭṭaka* (indeterminate equations of first degree), *varga prakṛti* and *cakravāla* (indeterminate equations of second degree). In the latter section, Bhāskara teaches the application of the Sutras stated earlier.

While discussing *khahara*, *kuṭṭaka* and *cakravāla*, there have been valuable contributions by *Bījagaṇita's* commentator, Kṛṣṇa Daivajña.

In this paper, the salient features of the text along with *Vāsanā*, the author's auto commentary and Kṛṣṇa's ideas are being explained.

Sūryaprakāśa of Sūryadāsa – A Review

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Bhāskara II of 12th Cent. A.D. was one of the first to write separate texts on arithmetic and algebra, *viz.*, *Līlāvatī* and *Bījagaṇita*. The early Indians regarded algebra as a science of great importance and utility. So, Bhāskara says: *ekam eva matir bījam* (i.e.) “intelligence alone is algebra”.

The distinction between arithmetic and algebra, to some extent can be found in their special names. While arithmetic (*vyakta gaṇita*) deals with mathematical operations, algebra (*avyakta gaṇita*) deals with determination of unknown entities.

In olden days Gurukula was the way of schooling and the texts contained enunciation in the form of cryptic *sūtras* for the sake of brevity and retention. To explain them commentaries arose. These commentaries made their own contribution to the development of Indian mathematics. The *Bījagaṇita* of Bhāskara also has a lot of commentaries. Of its many commentaries *Sūryaprakāśa* of Sūryadāsa (partially) and *Bījapallava* of Kṛṣṇa Daivajña have been published.

Sūryadāsa was a versatile genius who wrote on a wide variety of topics. Besides being an astronomer, he was also a poet. His commentary is the first known commentary on *Bījagaṇita* of Bhāskara. Among his works the *Sūryaprakāśa* (1538 A.D.) and the *Gaṇitāmṛtakūpika* (1541 A.D.) are commentaries on Bhāskara's works.

Sūryadāsa's use of the approximate square-roots of *karaṇīs* to demonstrate the validity of the rule concerning the sum and difference of two *karaṇīs* in Bhāskara's verse is a novelty. It is noteworthy that Sūryadāsa's verse on Śrīdhara's formula for quadratic equation is quite different from that of the other commentator, Kṛṣṇa Daivajña.

This paper, based on the printed text of Pushpa Kumari Jain (up to *kuṭṭakādhikāra*) and also the two manuscripts I.O. 1533a and *Prājña Pāṭhaśālā Maṇḍala*, Wai. 9777/11-2/551, reviews the commentary of Sūryadāsa on *Bījagaṇita*.

Bhaskaracharya and Varga Prakriti
The equations of the type $ax^2 + b = cy^2$

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Bhaskaracharya was the legend of his times. He wrote his theories in his granth ‘Siddhantasiromani’. This granth has four sections. They were Lilavati, Bijaganita, Grahaganita and Goladhya. In his work of Bijaganita, Bhaskaracharya have given some special methods to solve problems in arithmetic especially in Algebra. For some problems he has given the method but not the exact solution. So, some researchers have criticized his work saying that a further explanation is required and simply ignored the way to exact solution while some of them have given wrong illustrations for different methods.

In this paper we explain and discuss about Bhaskaracharya's work on Vargha Prakruti to solve indeterminate quadratic equation of the type $ax^2 + b = cy^2$ where x and y are variables while a, b, c are constants. This method is given in shloka 70 and 71 by Bhaskaracharya in his book Bijaganita. Bhaskaracharya calls the letter 'a' as 'Prakruti' and as 'Prakruti', the Nature is fixed, the same way the value of 'a' does not change throughout the method for particular problem. We further discuss that if one simple solution of the equation is obtained then using the method of Vargha Prakruti infinitely many integral solutions can be obtained iteratively for the given equation. Also, we obtain that the square-root of particular natural number 'a'- Prakruti which is not a perfect square, using this method iteratively, correct up to some places of decimal. This method is also applicable to solve the famous Pell's equation $ax^2 + 1 = y^2$.

Bhāskaracārya's mettle in dealing with infinitesimal quantities

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Bhāskarācārya is regarded as one of the best mathematicians of his time. His works in the field of mathematics are quoted by many famous authors belonging to later times. He is accredited with transforming the landscape of Indian mathematics by dealing with complex problems such as $Nx^2+1=y^2$ (now called as Pell's equation) and proposing ways for solving them. His text *Siddhānta Śiromaṇi* is regarded as *Magnum opus* in the field of Astronomy.

Siddhānta Śiromaṇi deals with finding the position of sun and planets at a desired time and location. It also deals with finding eclipses, heliacal rising and settings, position of moon's horns etc. The need for finding planetary positions is of paramount importance as Vedic texts instructs one to perform rituals on auspicious times depending upon the zodiacal position of planets and on occurrence of special phenomena such as eclipse.

In his text he has shown instances where he used certain techniques that resemble methods employed in calculus to study the behavior of functions to infinitesimally small changes. *Spaṣṭādhikāra* is one chapter where, after construction of sine tables, the author employs a method of interpolation to find the sine value of angle which is intermediate between two entries in the table.

Spaṣṭādhikāra deals with finding correct position of planet given that mean position is already obtained (from *Madhyamādhikāra*). In this chapter, the author describes construction of sine tables which is necessary for the calculation of correct position of planets.

While sine values of specific angles such as 30º, 45 º, 60 ºcan be derived easily, one needs a working knowledge of various trigonometric relationships to find out the Sines for angles which take finer values, like 15º,19º etc. He has also used quadratic interpolation which goes beyond the 'rule of three' to find closer approximations to sine values for angles not found in table.

In this paper an attempt is made to show Bhāskarācārya's mettle in handling smaller quantities which is rudimentary for calculus and interpolations.

Bhaskaracharya's Kuttak Method

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Bhaskaracharya was the legend of his times. He wrote his theories in his granth 'Siddhantasiromani'. This granth has four sections. They are Lilavati, Bijaganita, Grahaganita and Goladhya.. Of these Lilavati and Bijaganita are two books related to Mathematics and Algebra respectively. In both the books Bhaskaracharya have given examples and methods to solve problems algebraically and numerically. Some problems and the methods to solve are so simple that even a high school student can easily grasp it and understand. But some of them are so complex that even a scholar in the subject finds it difficult to analyse and see the solution for it.

In this paper we study and discuss the Kuttak Method. The equation of the form $ax \pm b = cy$ is called Kuttak where x and y are variables while a, b, c are integral constants. Bhaskaracharya has called 'a' as *Bhajya* ,'b' as *Har* , 'c' as *Shepak* ,'x' as *Gun* and 'y' as *Labhadi* . To solve this equation means to obtain least positive integral solution of this equation. In his book Lilavati, in verses numbered from 235 to 239 he has discussed the method for the general case and in verse 234A Bhaskaracharya has given the method to solve particular equation $100x \pm 90 = 63y$. In this paper we apply and prove Kuttak Method to solve the equations of the type $ax \pm b = cy$. Also we study and discuss the Kuttak Method to solve the particular equation $100x \pm 90 = 63y$.

The Critical Study of Algorithms in Karaṇīṣaḍvidha

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The contents of Bhāskaracārya's script 'Bījagaṇita', excluding Cakravāla, enjoy lesser world accolade. The forth chapter in the script titled 'Karaṇīṣaḍvidha' is one such example. Here the author, for the first time in the history of Indian mathematics, had discussed six operations on Karaṇī (Surds or Irrational numbers) namely Addition, Subtraction, Multiplication, Division, Square and Square roots in seven aphoristic Samskṛta verses, which hide as much as they reveal even though supported by nine illustrations.

This paper first extends the scope of the operations from integer to rational surds and then set out clear-cut definition and algorithm for each of them by paraphrasing the essence of the original verses, their translations and commentaries. The validity criterion and the preconditions for the existence of the operation are also discussed while furnishing the corresponding rationale and the illustrations.

V THE SIDDHĀNTAŚIROMAṆI, GAṆITĀDHYĀYA

Grahagaṇitādhyāya of Bhāskarācārya's *Siddhāntaśiromaṇi*

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Siddhāntaśiromaṇi composed around 1150 CE by Bhāskaracārya II is one of the most comprehensive treatises on Indian astronomy. It incorporates most of the astronomical knowledge prevalent in India in Bhāskara's times, and makes significant additions to it. Bhāskara's own *Vāsanā* explains the statements and algorithms in the verses, and the *Upapatti*s prove/demonstrate most of the results. In this presentation, we confine our attention to the *Grahagaṇita* part of *Siddhāntaśiromaṇi* (*Golādhyāya* being the other part). In particular, we discuss the chapters on *Madhyamādhikāra* (mean longitudes), *Spaṣṭādhikāra* (true longitudes), and *Tripraśnādhikāra* (the three questions, namely, direction, time, and space, essential for discussing the diurnal problems), in some detail.

At the very outset, Bhāskara pays tribute to the (Indian) astronomers preceding him, especially Brahmagupta, whom he calls *Gaṇakacakracūdāmaṇi* , and states that he (Bhāskara) has made improvements upon their results and understanding at several places in the text, and exhorts the reader to go through the whole work to appreciate it. Bhāskara lives up to his promise, and we highlight some of the important features of the text.

The knowledge of 'time' is the very essence of astronomy. The nature of time and the various units of time, and their significance are spelt out. The determination of the planetary parameters including the revolution numbers, using a *golayantra* is explained in great detail. The epicycle and eccentric circle models for the true longitudes of the planets are described. We also have a detailed account of the concept of 'instantaneous rate of motion', which involves ideas of calculus, especially the derivative of the sine function.

Bhāskara had a deep understanding of the rule of three (*trairāśika*), the application of which is ubiquitous in the diurnal problems, through similarity of the relevant triangles . We give a few examples of the nontrivial use of *trairāśika* in some problems. Alternate methods for derivation of results (some of them, very elegant) are provided, wherever possible.

Siddhāntaśiromaṇi had a significant impact on the subsequent works by Indian astronomers, such as those by the Kerala astronomers Nīlakaṇṭha Somayāji, and Jyeṣṭhadeva, who carry forward the tradition with more systematic treatments of problems, and more exact results.

Dik, desa and kāla in Bhaskarācharya's siddhāntic work – Siddhāntaśiromoni

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Bhaskārāchārya was a great mathematician and astronomer of 12th century AD. He has a lot of works like Lilavati, Siddhāntaśiromoni etc. and his own commentary on Siddhāntaśiromoni, Vāsanābhāṣya. The Lilāvati which is based on Brahmagupta's Brāhmasphutasiddhānta, Srīdhara's patigaṇita and Āryabhata II's exhibits a profound system of arithmetic and also contains many useful propositions in geometry and arithmetic. This work was translated into Persian by Fyze in 1587 C E by the direction of the emperor Ākbar. In his master work Siddhāntaśiromoni, we get evidence of his knowledge of trigonometry including sine table and different relations among the three functions known as jyā, kojyā and utkramajyā. This paper intends to study the chapter triprasnādhikāra containing 109 verses deals with Indian methods of spherical trigonometry, gnomonics or observation with the help of a gnomon etc. Bhāskarāchārya acknowledged all his predecessors about the knowledge of astronomy which are rare at that time. In the late medieval period Siddhāntśiromoni became very popular. Copies of manuscripts are available in India and abroad. In the triprasnādhikāra chapter we find a usage of 'golayantra' or armillary sphere which helped the astronomers to solve the diurnal problems.

Heliacal Rising of Stars and Planets in Bhāskara's Works

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In the present paper we discuss briefly the phenomenon of heliacal rising and setting of stars and planets. The heliacal rising and setting of stars and planets is an important event discussed in all classical *siddhāntas* under the chapter "*Udayāstādhikāra*". Even in modern astronomy great importance is given to this topic. Bhāskara II discusses this phenomenon in detail in his *Siddhāntaśiromani* as well as *Karaṇakutūhala*.

The importance of stars rising and setting, apart from its religious significance, lies in its becoming *circumpolar* for different latitudes during different periods, usually in intervals of thousands of years. When a star or a planet is close to the sun, within the prescribed limit, the concerned body is not visible due to the sun's effulgence; this is called the '*Heliacal setting'* of the concerned star or planet. After a few days when the heavenly body is outside the prescribed angular distance from the sun it becomes visible

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and remains so for quite a few days. This visibility is called '*Heliacal rising* '. A star is said to be circumpolar, when viewed from a particular terrestrial latitude, either does not set at all (i.e. always visible) or does not rise at all (i.e. always invisible) for several years. The *udayāmśa* and *astāmśa* are different for different heavenly bodies and even for particular star these vary with the terrestrial latitude. Further, due to the precession of the equinoxes the rising and setting points for any given place change, though slowly, over centuries.

In this paper, we show how stars which become *circumpolar* for extreme latitude attained some status after a couple of thousands of years for a lesser latitude. Thus, the course of *circum-polarity* of stars moves reducing the terrestrial latitude successively until it reaches a limit.

Vyatipāta and *Vaidhṛti* (Parallel Aspects) as discussed in Bhāskara's Works

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In the present paper we discuss briefly an interesting astronomical event called "Parallel phenomenon" or "Parallel aspect" in astro- sciences. Besides explaining this phenomenon we provide interesting historical Indian references recorded in classical astronomical texts as also in some old inscriptions.

When the Sun and the Moon in the course of their apparent motion, as observed from the earth, are placed equidistant with respect to the celestial equator, the phenomena of 'parallel aspect' is said to occur, if both the Sun and the Moon are on the same side of the celestial equator (*viṣuvadvṛtta*)-both to the north or both to the south, then the phenomenon is called *Vaidhṛti*. This means the declinations (*krānti)* of the two bodies are equal, both in direction and magnitude. On the other hand if the bodies are equidistant on opposite sides of the celestial equator they are said to be in *Vyatipāta* phenomenon, in that case their declinations are equal in magnitude but opposite in directions. In classical Indian astronomical texts generally a section is devoted to a deep study of these two phenomena under "*Pātādhikāra".* Importance has been given to this parallel phenomenon ever since the time of *Vedāṅgajyothiṣa,* the earliest Indian astronomical text in Sanskrit. In the ancient Jain tradition also this astronomical phenomenon is recognized, for example in the famous *prākṛt* text *Jyotiṣkaraṇḍa.*

In our paper we are highlighting (i). Bhāskara's procedure, (ii). an example given by Sumati Harṣa (fl.1617 CE) in his commentary on *Karaṇa-kutūhala* and (iii). historical references to *Vyatipāta* and *Vaidhṛti* in medieval Indian inscriptions.

Solar and Lunar Eclipses in Indian Astronomy

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In ancient and medieval Indian astronomical texts (*siddhāntas, tantras and karaṇas*) great importance is given to the phenomena and computations of eclipses, (*grahaṇa, uparāga*). The Indian astronomers used to put to test their theories and procedures, especially in respect of the Sun and the Moon, on the occasions of eclipses and conjunctions (*nirīkṣya grahaṇa grahayogādiṣa*…..). As and when disagreements occurred between the observed and the computed positions, the great savants of Indian astronomy revised their parameters and when necessary even the computational procedures.

In the present paper, the procedures for the computation of lunar and solar eclipses based on Bhaskara II's works are explained. The procedures are illustrated with actual instances of eclipses. The main parameters required in the procedures are: (i)The instant of conjunction or the opposition of the Sun and the Moon, (ii)True daily motions of the two bodies, (iii)The angular diameters of the Sun and the Moon for solar eclipse and those of the Moon and the earth's shadow cone (*chāya bimba*), (iv)Moon's node (*pāta, Rahu*), (v)The Moon's latitude (*ś*ara) at the instant of conjunction or opposition as the case may be.

In our paper, we have suggested an improved siddhantic procedure (I S P) by which we obtain the instants of circumstances comparable to those by modern procedures.

Bhāskara II and *Kṣayamāsa*

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What characterises the Indian calendar after the development of astronomy is the practice of the omitted month (*kṣayamāsa*), which is the opposite case of intercalary month (*adhimāsa*): a lunar month is omitted from the naming. The occurrence of omitted month is quite seldom, but very theoretical nature of Indian astronomy required to take it into account. In China and Japan possibility of the omitted month was known around the beginning of the 19th century, but it was not put into practice.

As far as I know, the first Indian astronomer who discussed the problem of the *kṣayamāsa* is Bhāskara II. In his *Siddhāntaśiromaṇi* (*Grahagaṇitādhyāya* 6.6-7) he says:

asaṃkrāntimāso 'dhimāsaḥ sphuṭaḥ syād
dvisaṃkrāntimāsaḥ kṣayākhyaḥ kadācit /
kṣayaḥ kārttikāditraye nānyataḥ syāt

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tadā varṣamadhye 'dhimāsadvayaś ca //6//

gato 'bdhyadrinandair (974) mite śakakāle
tithīśair (1115) bhaviṣyaty athāṅgākṣasūryaiḥ (1256) /
gajādryagnibhūbhis (1378) tathā prāyaso 'yam
kuvedendu (141) varṣaiḥ kvacid gokubhiś(19) ca //7//

The (lunar) month which has no *saṃkrānti* would be the true additional month. Sometimes (when there would be) a month which has two *saṃkrānti*s, (then it is) called 'omitted' (*kṣaya*). *Kṣaya* would be in the three months beginning with Kārttika, and not in other (months). Then there are two *adhimāsa*s within the year. When 974 years measured in the Śaka Era expired (there was a *kṣayamāsa*), and it will occur in Śaka 1115, 1256, and 1378. Thus this is mostly in every 141 years and sometimes (after) 19 years.

In my communication I would like to check the validity of his words using my pancanga program based on the *Sūryasiddhānta*. I also refer to the *Śiromaṇiprakāśa* of Gaṇeśa (1600-1650) from Nandipura, Gujarat, who, quoting the verses from the work of Gaṇśadaivajña (b. 1507), gives a further list of Śaka years when *kṣayamāsa* occurred or would occur.

VI THE SIDDHĀNTAŚIROMAṆI, GOLĀDHYĀYA

Vāsanābhāṣyas of Bhāskarācārya: Explaining and Justifying the Results and Processes in Mathematics and Astronomy

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Apart from composing the celebrated textbooks of Indian mathematics and astronomy, viz. *Līlāvatī, Bījagaṇita* and *Siddhāntaśiromaṇi*, Bhāskarācārya also composed the *Vāsanābhāṣyas*, commentaries which have acquired the status of canonical expository texts as they present detailed explanations and justifications for the results and processes outlined in the basic works of mathematics and astronomy.

In his *Vāsanāvārttika* (c. 1621), Nṛsiṃha Daivajña mentions that Bhāskarācārya first composed the commentary *Vivaraṇa* on the *Śiṣyadhīvṛddhida* of Lallācārya. Apart from presenting *vivaraṇas* or explanations, this commentary also presents *upapattis* or justifications. On the other hand, the *Vāsanā* commentaries of Bhāskarācārya on *Līlāvatī* and *Bījagaṇita* largely confine themselves to presenting the solutions of the *udāharaṇas* or examples presented in these texts, which deal with arithmetic and geometry (*pāṭīgaṇita*) and algebra, respectively. There are however quite a few instances where these commentaries do present important explanations and insights. It is in his *Mitākṣarā* or the *Vāsanābhāṣya* on *Siddhāntaśiromaṇi* that Bhāskarācārya presents detailed explanations as well as *upapattis* or justifications. This commentary is indeed a seminal text which also includes many illuminating discussions on the methodology of the science of astronomy in the Indian tradition.

In our presentation, we shall discuss some of the important *upapattis* or justifications presented by Bhāskarācārya in his commentaries. In particular, we shall focus on the *Golādhyāya* (the section on Spherics) of *Siddhāntaśiromaṇi*, which itself is meant to clarify the methods of *Grahagaṇita* or computation of the motions of the planets. As we shall see, the *Vāsanābhāṣya* on *Golādhyāya* includes many interesting demonstrations and methodological discussions which serve to illustrate the approach of Indian astronomers in the middle of the twelfth century.

Some Aspects of the Commentary *Vāasanāvārtika* of Nṛsiṃha Daivajnña on *Vāsanābhāṣya* of Bhāskarācārya

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Siddhāntaśiromaṇi of Bhaskara-II is one of the greatest works in the tradition of Indian astronomy. It can be taken to represent the knowledge of astronomy prevalent in India at the time of its composition (c. 1150 CE) plus Bhāskara's own improvements and innovations.

The text consists of two parts, namely, *Grahagaṇita* (Planetary computations) and *Golādhyāya* (Spherical astronomy). It is significant that Bhāskara himself has written a detailed commentary *Vāsanābhāṣya* on this work. Some of the other important commentaries on the work are: *Vāsanā-Vārtika* by Nṛsiṃha Daivajña (a super-commentary on a *Vāsanābhāṣya*) and *Marīci* by Munīśvara. At several places, these commentaries go beyond the explanations in Bhāskara's *Vāsanābhāṣya*, and provide insights into the Indian understanding of astronomy at the times when they were composed.

Nṛsiṃha Daivajña, the author of *Vāsanāvārtika* was from Maharashtra. His father's name was Kṛṣṇa Daivajña. It is to be specially noteworthy that, in *Vāsanāvārtika*, Nṛsiṃha examines the conventional/technical words (*pāribhāṣika-śabdas*). In this paper we will present some interesting highlights of *Vāsanāvārtika*, especially discussions on the nature of planetary theory, determination of astronomical parameters such as revolution numbers of the planets, their apogees, perigees etc. which are mentioned in the *bhagaṇādhyāya* of *Madhyamādhikāra*. For instance, Bhāskara gives a method to find the length of the year through observations of the direction of the sunrise, and the change in it after 365 and 366 days. The value for the length of the year stated by him corresponds to the sidereal year, whereas it is the tropical year which is determined through his method. Nṛsiṃha clarifies the situation in his *vārtika*. We would also touch upon the discussion of the "Greek (*yavana*) theories" by Nṛsiṃha.

An Application of the Addition Formula for Sine in Indian Astronomy

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It is well known that Bhāskara is the first Indian astronomer who mentioned clearly of the addition formula of sine and cosine in the *Jyotpatti* section of his *Siddhāntaśiromaṇi*.

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Bhāskara himself did not give any rationale but several following astronomers explained and utilized the formula. We recently found an application of the formula given by famous Kerala astronomer-mathematician Mādhava and rationalized by Nilakantha Somasutvan in his *Āryabhaṭīya-Bhāṣya* composed in the early 16th century. The formula is for adding or subtracting two sines which are not in the same plane, and utilized for calculating the declination of a planet when it deflects from the ecliptic. In my speech I will introduce this formula given by Mādhava and its rationale by Nīlakaṇṭha.

Jyotpatti

Anant W. VYAWAHARE & Sanjay M. DESHPANDE

Bhavabhuti Mahavidyalaya, Amagaon

Bhaskaracharya , (Bhaskar II, 1114- 1185 AD), was one of the great mathematicians, India has produced. His text “Sindhant Shiromani” (SS) was treated as the base of the further research oriented results by the mathematicians after him. SS contains two parts:, *Goladhyaya and Grahaganit*. Jyotpatti is the last chapter in Goladhyaya. Jyotpatti consists of 25 *Shloka* (stanzas), all in Sanskrit language. It is a general impression that SS contains *Lilaavati* and *Beejganit* also, but that is not so.

2. Jyotpatti deals with trigonometry. This was a milestone in developing geometry in India. Jya means sine and Utapatti means creation. Hence the name Jyotpatti. Jya and Kojya(or kotijya) stand for the Rsine and Rcosine ratios. The trigonometry developed by Bhaskara II is based on a circle of radius R, and not on a right angled triangle as taught in schools.

3.After defining Jya, Kotijya and Utkrama (verse) jya.etc, Bhaskara obtains these ratios for the standards angles of 30,45, 60, 36 and 28,(all in degrees) by inscribing a regular polygon in a circle of radius R. Bhaskara called these angles as *Panchajyaka.* Not only this, Bhaskara developed these results for addition and subtraction of two angles. This was further developed for the similar results for the multiple angles. . Bhaskara compares jya and kotijya with the longitude and lateral threads of a cloth.

4. **Contents in Jyotpatti** (Only a few are mentioned here)

1. R jya 45 =$R/\sqrt{2}$, and other similar R jya values. (all in degrees).
2. R Jya 36 = 0.5878 approx.
3. Sn = side of a regular polygon of n sides = D sin π/n, D is the diameter of circle in which polygon is inscribed.
4. Derivation of formulae for sin(θ +Φ) and cosine(θ +Φ) called as *samas bhavana and antar bhavana*
5. *Bhuj Koti karna nyaya* , that is, a theorem of square of hypotenuse(now known as Pythagoras theorem).
6. Concept of derivatives, that is, $\partial(\sin\theta)$ =($\cos\theta)\partial\theta$, etc., Concept of Rolle`s theorem.

5. **Comments**
   1. Jya and Kotijya are defied on a unit circle, using diameter and a chord.
   2. These ratios are defined on the arc of the circle of radius R subtending angle θ at the center of the circle.(and not angles directly.)
   3. Initially, graphical methods are applied.
   4. *Tatkalika* methods(instantaneous) introduced for motion of planets.
   5. Many results have been derived using *Goladhyay*. Without reading *Goladhyaya*, Jyotpatti will be difficult to understand.

Astronomical Instruments in Bhāskarācārya's Siddhāntaśiromaṇi

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Following the model set up by Brahmagupta, Bhāskarācārya includes a chapter on instruments in the *Golādhyāya* of his *Siddhāntaśiromaṇi* and describes there ten instruments for observation and time-measurement, namely *Gola, Nāḍīvalaya, Yaṣṭi, Śaṅku, Ghaṭī, Cakra, Cāpa, Turya, Phalaka, Dhī.* The *Gola-yantra* (armillary sphere) was also described in an earlier chapter entitled *Golabandhādhikra.* Among these instruments, the *Phalaka-yantra* appears to be his own invention.

Bhāskara's attitude towards these instruments is rather ambivalent. On the one hand, he is very pragmatic and dismisses as unnecessary certain specifications given by his predecessors about some instruments, on the other hand he adds three varieties of self-propelling instruments (*svayaṃvaha-yantras*) to his repertoire, not because they serve any purpose in the *Golādhyāya* but just because others have described them.

In this paper, we propose to discuss Bhāskara's attitude towards instrumentation in general, examine in detail the various instruments described by him, note the valuable comments made on some of these instruments by Nṛsiṃha Daivajña in his commentary *Vsanāvārttika,* trace the historical antecedents of these instruments and relate these instruments to specimens which are extant today.

Jñānarāja's Critique of Bhāskarācārya's *Siddhāntaśiromaṇi*

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Jñānarāja composed the *Siddhāntasundara*, an astronomical treatise belonging to the *saurapakṣa* school of Indian astronomy, around 1500 CE. It was the first major *siddhānta* to appear after the *Siddhāntaśiromaṇi* of Bhāskarācārya from 1150 CE. The *Siddhāntaśiromaṇi*, well known for its depth and comprehensiveness, was hugely influential and normative at the time of Jñānarāja, and as such, it was a treatise that Jñānarāja had to engage with. Jñānarāja, writing at the beginning of the early modern period, was perpetuating an ancient tradition of astronomy while addressing the needs of his times and highlighting his own contributions. While hugely influenced by the *Siddhāntaśiromaṇi*—especially by Bhāskarācārya's idea of *vāsanā* (that is, demonstration)—Jñānarāja was also critical of some of the assumptions and formulae of Bhāskarācārya. For example, Jñānarāja rejects Bhāskarācārya's argument that the earth is its own support and presents his own argument that the support is derived from divine beings. Jñānarāja also rejects a formula from the *Siddhāntaśiromaṇi* due to breaking an equinoctial day, where it produces the mathematically meaningless result 0/0. This talk will focus on Jñānarāja's use and critique of Bhāskarācārya's *Siddhāntaśiromaṇi*. Attention will be paid to the different times and milieux of the two astronomers in shaping their treatises.

VII THE KARAṆAKUTŪHALA

Importance of Karaṇakutūhala as an algorithmic Handbook

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Bhāskara II's greatness lies in making mathematical procedures direct and simple. Needless to say, his language is again simple, poetic and almost irresistible. Besides his famous astronomical treatise *Siddhānta śiromaṇi* and the two mathematical works *Līlāvatī* and *Bījagaṇita,* Bhāskara II composed a very useful and popular astronomical handbook called *Karaṇakutūhala* (*KK*).

Bhāskara composed *KK* in 1183CE when he was 69 years old. The epochal date of this text is February 24, 1183CE(Julian), Thursday and the time is the mean sunrise at Ujjayinī. This tract is also known as *Grahāgama Kutūhala*. This handbook enjoyed remarkable popularity for centuries, mainly in Maharashtra and much of north India among the *Pañcāṅga* makers belonging to the *Brāhmapakṣa*. Even now, the practitioners of this school of astronomy do use *KK*. The popularity of the text is evident from the availability of a large number of manuscript copies in the libraries and private collections.

In obtaining the true positions of the heavenly bodies from the mean, besides the equation of the centre (*mandaphala*) due to the eccentricity of the orbit, the other important corrections applied are the following: (i) *Deśāntara saṃskāra* due to the difference in longitudes of the given place and the central meridian (Ujjayinī); (ii) *Cara saṃskāra* due to the difference of the local latitude from the equator (*Laṅkā*) and (iii) *Bhujāntara saṃskāra*.

The *bhujāntara* correction is to obtain the true instant -- from the mean -- of midnight (or the noon) and, is called 'the equation of time'. This correction is made up of two components: (i) the one due to the earth's *eccentricity*, in terms of the sun's equation of the centre; and (ii) the other due to the obliquity of the ecliptic with the celestial equator. The latter correction is called *udayāntara saṃskāra*. Bhāskara II gets the credit of explaining this *udayāntara* correction adequately and adopting it in his procedural scheme.

Some aspects of the procedures for (i) true positions of the heavenly bodies, (ii) lunar and solar eclipses, (iii) conjunction phenomena including occultations of stars and planets as also the transits of Venus and Mercury and (iv) heliacal rising and setting of planets and stars are highlighted in the present paper with illustrations of contemporary phenomena.

Siddhānta-Karaṇa Conversion
Some algorithms in the Gaṇitādhyāya of the Siddhāntaśiromaṇi and in the Karaṇakutūhala

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The relationship between treatises or *siddhāntas* and handbooks or *karaṇas* in Indian astronomy is an interesting one. Meant to facilitate computation for astronomical practice but nonetheless still committed to the parameters and techniques of a particular astronomical school, the genre of the *karaṇa* has yet to be thoroughly analyzed. The work of Bhāskarācārya, with the handbook *Karaṇakutūhala* closely tracking the methods in the *Gaṇitādhyāya* of the magisterial *Siddhāntaśiromaṇi* but revealing many ingenious modifications of them, will be examined here as a case study in the relationships of these two technical genres.

Making Tables from Text: From the Karaṇakutūhala to the Brahmatulyasāraṇī

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A twelfth century set of astronomical tables, the so-called *Brahmatulyasāraṇī*, poses some interesting challenges for the historian. While these tables exhibit a range of standard issues that numerical data typically present, their circumstances and mathematical structure are further complicated by the fact that they are a recasting of another work that was originally composed in verse, namely Bhāskara II's *Karaṇakutūhala* (epoch 1183 CE). We examine the many links between these two works and what they reveal about the computational procedures and techniques that practitioners from this period employed when executing and applying these astronomical algorithms.

Karankutuhalam (Solar and Lunar Eclipse)

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In 1114 a great mathematician, astronomer was born in Vijjada Vida in India, named Bhaskaracharya. In 1183, he has also contributed in Ancient Indian Astronomical research which is explained in detail in his small astronomical text Karan Kutuhalam.

In this book Bhaskaracharya has explained about computation of Lunar Eclipse (Chapter 4 Candragrahanadhikarah) and Solar eclipse (Chapter 5, Suryagrahanadhikarah). In this paper methods of determining the half duration, khagrasa , totality etc., are explained. The methods given in the text are applied to an example and it is shown how the results are close to those given in the modern ephemerides. The aksa and ayana valanam are explained and their algebraic sum called spasta valanam is calculated. The valanam is used in drawing the diagram of the eclipse. The effects of parallax on the longitude and the latitude of the moon, respectively called lambana and nati are considered at length. The methods explained in the paper are applied to the example of the solar eclipse which occurred on August 11, 1999. The beginning and the middle instants of the eclipse differ from the ephemeris values by just 5 minutes.

There are other ancient methods also to compute eclipse like Ganesa Daivajna's Graha laghavam (GL), Brahmagupta's Khanda Khadyaka (KK), Surya Siddhanta (SS). But in SS the position of the moon and Rahu are somewhat inaccurate, in GL method is very lengthy and in KK as we increase the number of iteration the only we get accurate timings. In Bhaskaracharya's method only true position of sun, moon and earth is required.

In the era without advanced computing instruments like computers, Bhaskarcharya has established accurate methods of determining these astronomical phenomena. All the algorithms, in the paper will be supported by computer programs and the paper also explains computation of the solar and lunar eclipse in future.

Transits and Occultations in Bhāskara's *Karaṇa-Kutūhala*

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In Indian classical astronomy, the *Siddāntic* texts have discussed in detail the phenomenon of conjunctions of the Sun, the Moon and the Planets, between any two of them as also with some bright stars like *Rohiṇī* (Aldebaran), *Citrā* (Spica) and *Makhā* (Regulus). In the present paper we have discussed the actual working procedure based

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on Bhāskara's astronomical works, for not only transits of Venus and Mercury but also of occultations of planets and stars. The result compared well with those of modern procedure.

The ancient and medieval Indian astronomers had the practice of revising and updating their parameters periodically by meticulously observing the phenomena of eclipses and planetary conjunctions. While the detailed working of planetary conjunctions is discussed by Bhāskara in the chapter devoted to *Grahayuti*. Although the transits of Mercury and Venus are not explicitly mentioned (since those could not be observed with naked eye), they are discussed as a part of *Grahayuti*. The phenomenon of transits comes under *Asta.* Bhāskara II discusses these phenomena together under planetary conjunctions.

As illustrations we present the case of occultations of *Rohiṇī* (Aldebaran) and also the lunar occultations of *Śukra* (Venus). Further a very interesting but rare '**close conjunction**' of *Guru* and *Śukra* is also presented in our paper.

Besides explaining Bhāskara's procedure, we have presented an *Improved Siddāntic Procedure* which yields accurate results.

VIII PERSIAN TRANSLATIONS OF BHĀSKARĀCĀRYA'S WORKS

Persian Translations of Bhāskara's Sanskrit Texts during the 16th –17th Centuries and their Impact in the following Centuries

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It is now well known that two mathematical works of Bhāskara II (b.1114) were translated into Persian. They are:

1. *Līlāvatī* translated by the famous scholar Abu'l Fayḍ Fayḍī (1547 –1596) in 1587 at the instance of Emperor Akbar (reign 1556 –1605). A large number of manuscript–copies are extant in the libraries of India and Pakistan. The text was published in Calcutta 1827, 1832 and 1854. Another Persian translation of *Līlāvatī* was done by Mednī Mal b. Dharam Narayan b. Kalyān Mal prepared for Emperor Aurangzeb 'Alamgīr. Fayḍī's Persian text was also translated into Urdu in 1844 by Sayyid Muḥammad Ghāfil (ms. in Aligarh) and by another Hindu scholar Debi Chand in 1855. The Sanskrit original was translated into English by J. Taylor and published (Bombay 1816), and by H. T. Colebrook (London 1817).

2. *Bījaganita* translated by 'Aṭā'ullāh Rushdī or Rāshidī (son of Aḥmad Ma'mār the architect of Taj Mahal) dedicated to Emperor Shāhjahān (reign 1628–1658) and composed in 1634-5. The English translation of the Persian text was composed by E. Strachey, London 1813. Manuscripts extant in libraries of Hyderabad (2copies) and one each of London, Munich, Paris, Banaras, Calcutta, Rampur and also of Deoband and Luchnow Madarsahs. Another Persian translation of the same , entitled '*Ajāz al–Ḥisāb,* was carried out by Muḥammad Amīn 'Alwī during the reign of Aurangzeb in 1662 ; ms. in Raza Library (Rampur).

3. As for Astronomy, Persian translation of *Karanakatūhala* of Bhāskara II by an anonymous translator is extant in the collection of Punjab University Library (Lahore). The scribe is Gul Muḥammad (ca. 17th c.). It consists of 70 ff. I have found and identified another Persian manuscript of *Karaṇakatūhala* in the Raza Library (Rampur) , consisting of 25 ff. in a codex. It is not dated, but from the dates used in the calculations I assume it to be a copy of 15th c. Neither Storey nor Pingree mention this Persian translation. A commentary on the Persian translation of *Karaṇakatūhala* is also extant. Its title is *Sharḥ Frankūhal(a)*, in the Punjab Public Library, Lahore (Pakistan). The author is unknown, but he states in the text that it was written in 1809 Vikrama (i.e., 1751 AD).

4. Bhāskara's most important work, *Siddhāntaśiromanī* (composed in 1150 AD), was translated into Persian in 1212 AH/1797 AD by Ṣafdar ʻAlī Khān bin Muḥammad Ḥasan Khān, who dedicated it to Arasṭū Jāh (d.1804), the prime minister of Hyderabad. The translator gives this information in the opening folio (1b) of his another work, *Zīj-i Ṣafdarī* , composed 1234 AH / 1819 AD). This *Zīj* is actually the translation of *Grahacandrikāgaṇitā* by Appaya, son of Marla Perubhaṭṭa (fl. ~1491).

The translations from Sanskrit into Persian were continued during the reigns of Akbar Shāh II (beg. 1806) and Bahādur Shāh (end of reign 1857). They were mostly concerning astrology and calendar calculations. In any case, the reception of Ancient Indian sciences and also scriptures were rendered into Persian fervently even during the crumbling stage of Mughal empire. This reception is in fact a great tribute to the building of a composite Indian culture by the Medieval Indian intelligentsia.

I wish to present in my talk details of the above-mentioned information.

The Persian Translation of Bhāskarāchārya's *Līlāwatī*

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Līlāwatī is Bhāskarāchārya's treatise on mathematics written in Sanskrit. Līlāwatī was translated into Persian by Abu'l-Fażl Faiżī (1547-1595 A.D.) who was the great poet (Malik-ush-Shu'arā) of Akbar's Court. According to its preface, this work was completed in 1587 A.D. This version has been printed under the title The Līlāvatī, a Treatise on Arithmetic in Calcutta in 1827 A.D. The preface of the Persian translation concludes with a legend on a daughter of Bhāskarā called Līlāwatī and on the circumstances which led to the composition of a book bearing her name. Different English translations of Līlāwatī have been published by John Taylor (1816), H. Colebrooke (1817) and by K. S. Patwardhan, S. A. Naimpally and S. L. Singh (2001). Colebrooke's translation of Līlāwatī is divided into 13 chapters. The book covers many branches of mathematics: arithmetic, algebra, geometry, trigonometry and mensuration. According to Faiżī's preface the book included three section; introduction, some rules and ending. Bhāskarā never gives any proof of his formulas but each chapter contains some examples. Some of the translated extracts contain exposition of the rules and of technical terms.

In this paper, we present a brief account of the Persian translation and we compare it with original Sanskrit based on its English translations. We also review some examples from different sections of this book.

The Persian Translation of the *Bījagaṇita* by ‘Aṭā’ Allāh Rushdī

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Bījagaṇita which literally means “mathematics (*gaṇita*) by means of seeds (*bīja*)” is a Sanskrit work on algebra written by the famous Indian mathematician and astronomer of the 12th century A.D., Bhāskarāchārya. The treatise generally deals with algebraic equations. General methods are introduced for the solution of indeterminate problems for linear, quadratic, cubic, and bi-quadratic equations by means of several examples.

The Persian translation of *Bījagaṇita* was accomplished by ‘Aṭā’ Allāh Rushdī ibn Aḥmad Nādir for Shāh Jahān in 1634 A.D. The work is divided into an introduction (*muqaddama*), subdivided in six chapters (*bāb*); and five books (*maqāla*) which contain several chapters and sections. In the foreword, the translator states that this treatise includes significant and practical rules which are not mentioned in *Līlāvātī*, Bhāskarā’s arithmetic text. It seems that the Persian translator hasn’t fulfilled a meticulous translation of the original Sanskrit text of *Bījagaṇita*; instead, he has executed a mixture of text and his own commentaries.

In this paper, we present a brief account of the Persian translation of *Bījagaṇita* according to some extant manuscripts and we compare it with the original Sanskrit text based on its English translation by H. T. Colebrooke. We examine the additions and ellipsis of the Persian translation and try to find according to which sources these additions are affixed.

IX PEDAGOGICAL IMPORTANCE OF BHĀSKARĀCĀRYA’S WORKS TODAY

Pedagogical Analysis of Lilavati

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Lilavati, a book written by Bhaskaracharya in 1150 is a master piece of mathematical treatises. Bhaskaracharya, literally means Bhaskar the teacher. One notices that various facets of a good teacher interspersed within the text. The entire text is written in poetic form with profuse use of alliterations, pun, metaphors, etc. At several places he addresses the reader as *sakhe* (a female friend), *bale* (a young girl), mitra (a male friend), etc. Moreover, he frames the questions taking the help of animals and bird like elephants, monkey, serpent, peacock, swans, bees, etc. He also makes use of mythological stories from Hindu epics like Ramayana and Mahabharata to frame question. Lilavati has a huge collection of problems relating to arithmetical calculations, algebraic equations and properties of geometrical figures. The author attempts to develop essential pre-requisites through a series of examples and provides necessary hints required to solve a given problem. Openness is the hallmark of Lilavati as the author suggests different ways of dealing with the problem and leaves it to the reader to use the most appropriate one.

As a part of a yearlong celebration of Bhaskaracharya's 900th birth anniversary the Vidya Prasarak Mandal, Thane has initiated workshops on Lilavati for school children. Since January 2014 a dozen workshops have been conducted both in rural as well as the urban parts of India. Apart from acquainting them with salient features of Lilavati these workshops were used to test the utility of Bhaskaracharya's pedagogy in teaching school mathematics. It has been noticed that the pedagogic techniques advocated in Lilavati are found effective in removing students' fear of mathematics, motivating them to handle an unknown situation and developing problem solving skills. Constructivism, as a philosophy of learning, has gained importance in recent days and is advocated as an effective method of teaching school subjects. The critical analysis of Lilavati shows that Bhaskaracharya has used this technique about 900 years ago. It is high time that we implement it for effective teacher pupil interactions. The paper will present the pedagogic analysis of Lilavati and discuss its relevance for the teaching of mathematics in 21st century.

Līlāvatī: A Pedagogical Innovation of Bhāskarāchārya

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Since the antiquity of Indian Mathematics till 12th century C.E. in the lineage of Baudhāyana to Brahmagupta, the name Bhāskarāchārya (Born on1114 C.E.) suggests a teacher of profound reputation. Apart from this traditional view, the epigraphical evidence of Patne inscription (1148 shake=1207 C.E.) proclaims that nobody dared to debate with the disciples of Bhāskarāchārya. In order to understand his time-tested method to produce such a retinue of brilliant scholars, I would like to study the pedagogical approach of this celebrated teacher as detailed in the Līlāvatī (first part of his mathematical work Siddhāntaśiromaṇi). In my paper, I will highlight his shift of teaching methodology from the erstwhile established techniques described in the Upaniṣads (namely Taitteriya and Bṛhadāraṇyaka) to his own novel methodology that is comparable to modern teaching skills.

Unlike his predecessors in various disciplines of scientific thought, Bhāskarāchārya added a personal touch to his practice. From the very outset of Līlāvatī, he interacted with his pupil by addressing them directly and it became increasingly personal as he dealt with the harder concepts. He began with the “young lady”; this application of gender equality in a work of 1150 C.E. is unprecedented. Further along he embraced the student as “friend” and finally acknowledged them as the de facto “Mathematician”.

This sincere intonation was certainly an innovation in the academic milieu of ancient India. This informal approach must have satisfied the psychological needs (like “need for belonging”, “need for achievement”, “need for social recognition” etc.) and motivated them to pursue their analytical course not only to determine the planetary motion but also to carry forward the legacy of their maestro; The summation of all this being a generation of accomplished mathematicians who were widely respected throughout the Indian technological elite of the day. We are here to learn what made them shine.

Contributor’s Note

The fundamental research question i.e., ’The innovative pedagogical approach of Bhāskarāchārya in Līlāvatī in comparison with modern teaching skill’ and methodology adopted for the given are original. And this statement is true to the best of my knowledge.

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Bhaskaracharya's *Lilavati* in the Age of Common Core State Standards

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After the publication of the Common Core State Standards for Mathematics (CCSSM, 2010) in the United States, 42 of the 50 U.S. states have adopted these standards in their school mathematics curriculum. The CCSSM (2010) is ambitious not only in terms of mathematical content in each grade level and how they develop over time across grade levels but also in terms of eight mathematical practices that are expected of every student irrespective of their grade level. The standards for mathematical content from grades K-8 have been categorized under 11 domains such as operations and algebraic thinking, fractions, and geometry. These topics are further explored at the high school level. The CCSSM expects that the students will develop computational fluency using a variety of strategies (Dacey & Polly, 2012). It also expects students to "make sense and persevere in solving problems" and "reason abstractly and quantitatively."

This paper will focus on how Bhaskaracharya's work, published more than 850 years ago in his *Lilavati*, has the potential to help students achieve some of the content domains and mathematical practices as outlined in the CCSSM. This ancient text, composed of many computational rules, also has the potential to motivate students to persevere in solving mathematical problems by making explicit connections between arithmetic operations and algebraic thinking and by relating mathematics to their everyday world and other content areas, particularly astronomy and literature. Even though *Lilavati* did not necessarily explain why the rules worked, most of these rules can be easily connected to modern algebraic principles. In fact some of these rules for complex calculations expressed in poetic language (verses) can be fascinating to students around the world.

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Siddhantasiromani: A Continual Guide for Modern Mathematics Education

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Bhaskaracharya's Siddhantasiromani, which served as a textbook of mathematics across India, is about 864 old now. In many respects it stands outside the time. That is, several ideas and methods propounded by Bhaskaracharya in it are still relevant. Close reading of Siddhantasiromani reveals that the approach adopted by Bhaskaracharya there is in line with the modern theory of constructivism in mathematics education. It also has element to support an emerging viewpoint in some quarters that the mathematics should be treated as an empirical science.

The modern theory commends that mathematical concepts should not be transferred mechanically by teacher to the students especially those in younger age group, say at the primary and middle school levels. Creating situations that would foster the meaning and underlying relations of the mathematical concepts is considered the key. Nevertheless, implementing this way of teaching in the classroom has its own problems. Recourse-like tracing the historical development of a mathematical concept is found more practical. Understanding intermediate forms in concept development reveals quite a few dimensions and student can see the construction process in forwarding that concept and given the final shape.

One distinguishing point of Siddhantasiromani is that it provides methods to solve the problems without employing the concepts of limit and derivative of Calculus explicitly. By translating those methods using suitable computer software the students can now get a better feeling of construction in mathematics. Further, a few instruments outlined by Bhaskaracharya in his Siddhantasiromani could be redesigned by students for extensive measurements of objects and thereby enjoy the process of learning by doing.

To that extent the paper will first review select modern theories of teaching mathematics at the school level. Next it would critically examine the writings in Siddhantasiromani and evaluate the approach set out by Bhaskaracharya. On that basis guidelines for imparting instructions and approaching mathematics in general would be proposed.

Legacy of Bhaskaracharya: A Pioneer of Modern Mathematics

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Bhaskaracharya represents the pinnacle of mathematical knowledge in the 12th century. He reached to an understanding of the number systems and solving equations, which could be achieved in Europe only after several centuries. Bhaskaracharya was the first to discover gravity, 500 years before Sir Isaac Newton. Bhaskaracharya wrote Siddhanta Siromani which was a colossal work containing about 1450 verses. His works are Lilavati (The Beautiful) on mathematics; Bijaganita (Root Extraction) on algebra; the Siddhanta Siromani which is divided into two parts: mathematical astronomy and sphere; the Vasanabhasya of Mitaksara which is Bhaskaracharya's views on the Siddhanta Siromani; the Karanakutuhala (Calculation of Astronomical Wonders) or Brahmatulya in which he simplified the concepts of Siddhanta Siromani; and the Vivarana which comments on the Shishyadhividdhidatantra of Lalla. Lilawati is an excellent example of how a difficult subject like mathematics can be written in poetic language. His work on Kuttak is an astonishing work of high intellect and supreme brain power. Kuttak is nothing but the modern indeterminate equation of first order. His legacy to the modern world include a simple proof of the Pythagorean theorem, solutions of quadratic, cubic and quartic indeterminate equations, solutions of indeterminate quadratic equations, integer solutions of linear and quadratic indeterminate equations (Kuttaka), cyclic Chakravala method for solving indeterminate equations, preliminary concept of infinitesimal calculus and notable contributions towards integral calculus as well as differential calculus. He has also suggested simple methods to calculate the squares, square roots, cube, and cube roots of big numbers. In Goladhyay, Bhaskar has discussed eight instruments, which were useful for observations. Bhaskara's Phalak yantra was probably a precursor of the 'astrolabe' used during medieval times. He was the champion among mathematicians of ancient and medieval India. The concepts and methods developed by Bhaskaracharya are relevant even today.

Contribution of Bhaskaracharya in Modern Mathematical concepts: An overview

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Indian culture has a vast heritage in every field. The subject mathematics also has a great cultural heritage in India. It is the most to correctly understand our cultural heritage and interpret it for outside world. Many Indian Mathematicians have immense contribution in developing the subject of Mathematics from ancient to medieval and modern mathematics as well. Among many Indian mathematician, the contribution of Bhaskaracharya is really noteworthy. Many modern mathematical concepts like differential calculus, trigonometry, Instantaneous Velocity were known to Bhaskaracharya before its inception.

This paper throws light on these modern mathematical concepts known to Bhaskaracharya and try to work out with the comparative study of the mathematician like Newton, Euler, Lagranges, Leibnitz and Galois, who were considered to be pioneer of these concepts.